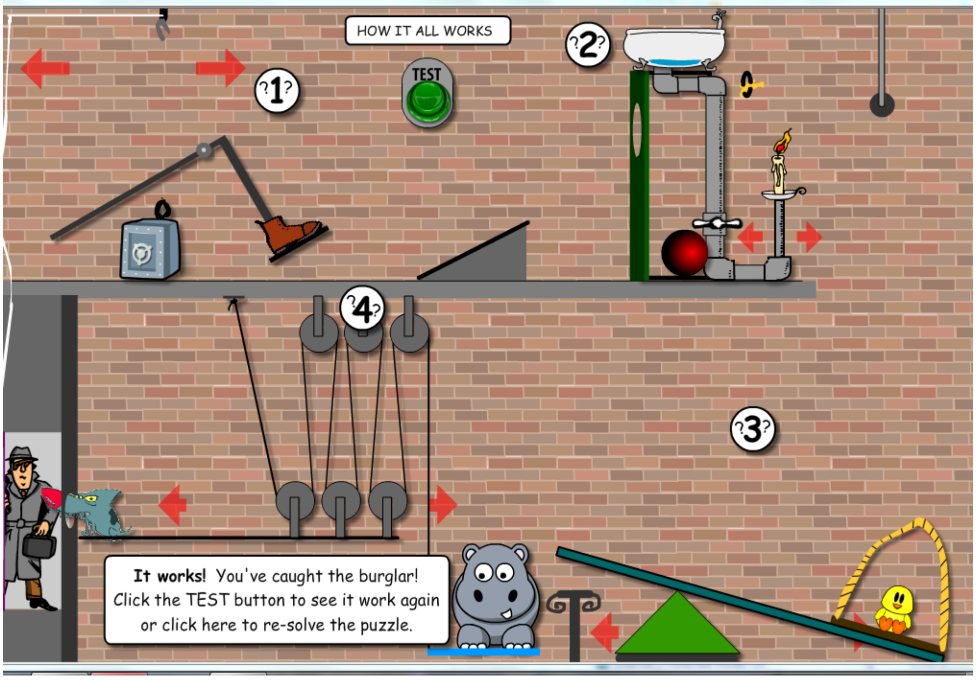
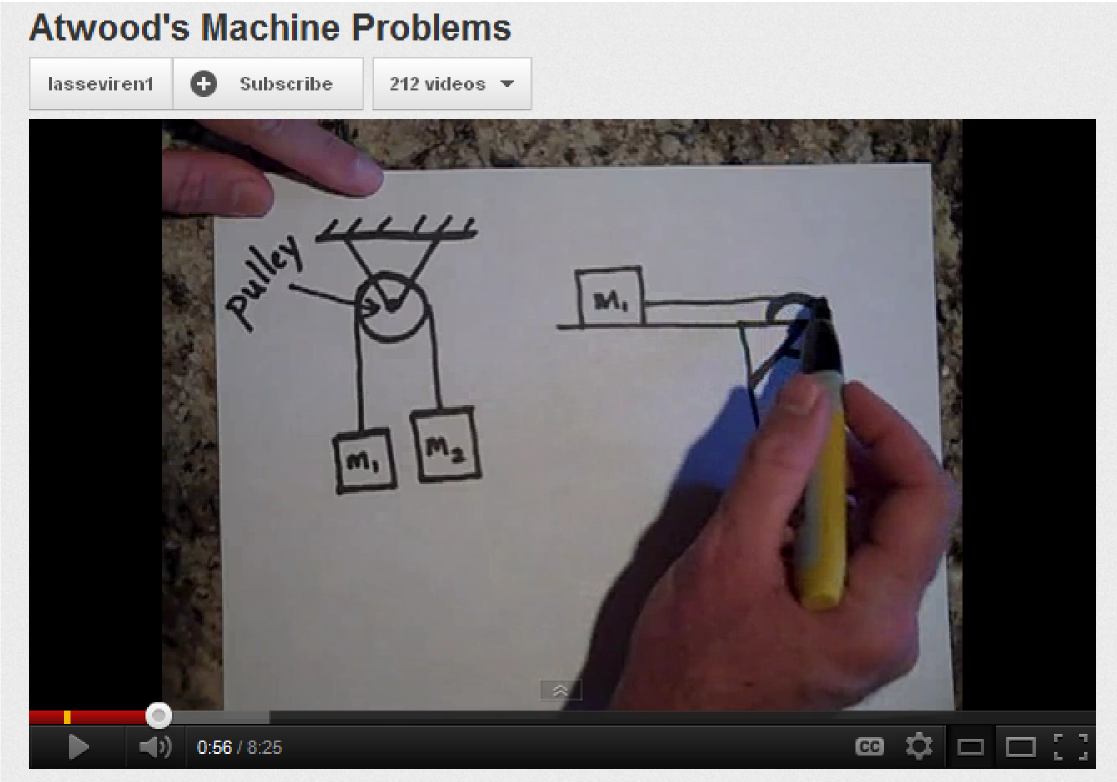
Physics 12

Pulleys

MiniScience.Com

Levers and Pulleys - Build a Rube Golberg Machine


HOW IT ALL WORKS
TEST
1
2
3
4
It works! You've caught the burglar!
Click the TEST button to see it work again
or click here to re-solve the puzzle.



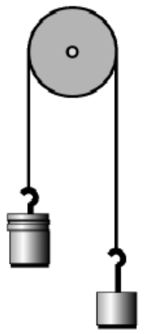
http://www.apphysicslectures.com

3:35 Part 2

Tension and Pulleys

Tension in a rope is the same everywhere in the rope, even if the rope changes direction (such as when it goes around a pulley).

ONE ROPE = ONE TENSION

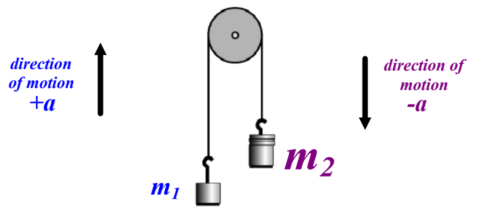


IMPORTANT NOTE

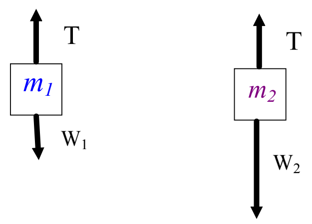
As the masses are connected, they will have the same acceleration, ***a***.

Atwood Machine

A block of mass m_1 and a block of mass m_2 are connected by a thin string that passes over a light frictionless pulley. Find the acceleration of the masses.



STEP 1: Free-body diagrams



STEP 2: Set up tension equations

$$T + (-W_1) = m_1 a \qquad T + (-W_2) = -m_2 a$$

$$T = m_1 a + W_1 \qquad T = -m_2 a + W_2$$

$$T = m_1 a + m_1 g \qquad T = -m_2 a + m_2 g$$

STEP 3: Set T expressions equal.

$$m_1 a + m_1 g = -m_2 a + m_2 g$$

STEP 4: Solve for acceleration, a.

$$m_1 a + m_2 a = m_2 g - m_1 g$$

$$a(m_1 + m_2) = m_2 g - m_1 g$$

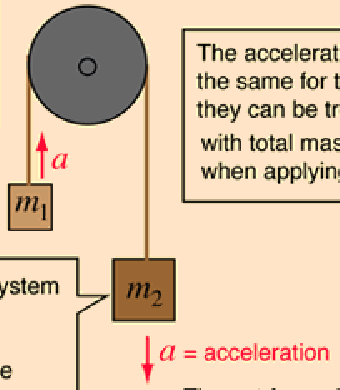
$$a = \frac{g(m_2 - m_1)}{m_1 + m_2}$$

Note: $m_2 > m_1$

Atwood's Machine

Frictionless case, neglecting pulley mass

As a more intuitive approach to Atwood's machine, consider that the total mass is accelerated by a force equal to the difference in the hanging weights.



The acceleration is numerically the same for the two masses, so they can be treated as a system with total mass $m_1 + m_2$ when applying F = ma.

Assuming m_2 to be larger, the system will accelerate in the direction indicated. The net force on the two-mass system is the difference in the weights

$$F_{net} = m_2 g - m_1 g$$

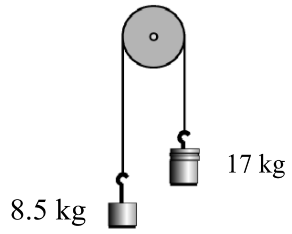
The net force divided by the total mass gives the acceleration.

The equation of motion for the two-mass system is then:

$$(m_2 - m_1)g = (m_1 + m_2)a \quad \text{or} \quad a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$$

Sample Problems - Pulleys

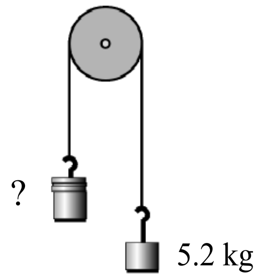
1. Find the acceleration of each object and the tension in the rope.



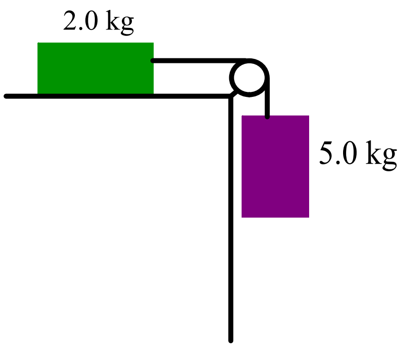
2. The smaller mass on an Atwood machine is 5.2 kg.

   a) Find the mass of the second object if the masses accelerate at 4.6 m/s^2.

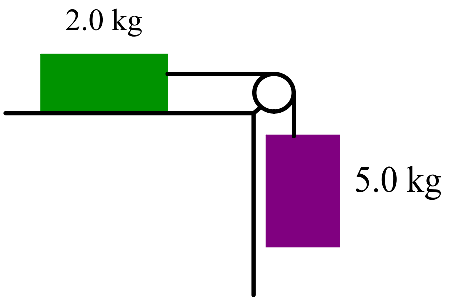
   b) Find the tension in the rope.



3. A 2.0 kg block and a 5.0 kg block are connected as shown. Find the tension in the rope connecting the two blocks. Assume the horizontal surface is frictionless.

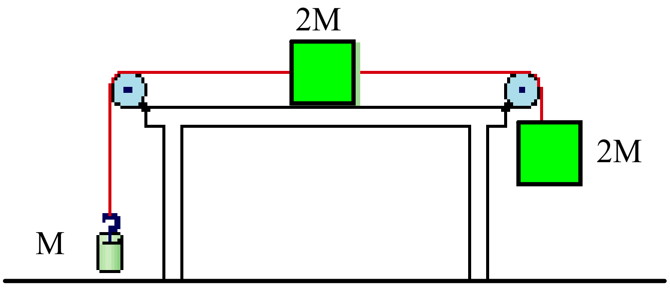


4. A 2.0 kg block and a 5.0 kg block are connected as shown. Find the tension in the rope connecting the two blocks. The coefficient of kinetic friction is 0.12.



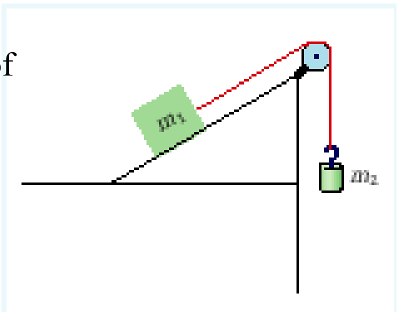
High Falutin' Pulley Problem

5. Three blocks are released from rest and accelerate at the rate of 1.5 m/s^2. If M = 2.0 kg, what is the magnitude of the force of friction that acts on the block that slides across the table?



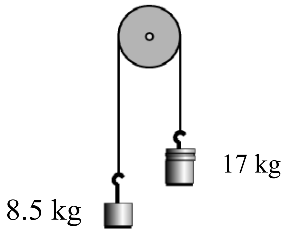
6. A block of mass 3.20 kg on a frictionless inclined plane of angle 30.0° is connected by a cord over a massless, frictionless pulley to a second block of mass 2.30 kg hanging vertically. What is:

a) the magnitude of the acceleration of each block?
b) the direction of the acceleration of the hanging block?
c) the tension in the cord?



Sample Problem

Find the acceleration of each object and the tension in the rope.



Answer

The acceleration of the 8.5 kg object is 3.3 m/s^2 up.
The acceleration of the 17 kg object is 3.3 m/s^2 down.

The tension in the rope is 1.1×10^2 N.

$F_{net} = m_1 a$ $\quad\quad$ $F_{net} = -m_2 a$

$T + (-w_1) = m_1 a$ $\quad\quad$ $T + (-W_2) = -m_2 a$

$T = m_1 a + W_1$ $\quad\quad$ $T = -m_2 a + W_2$

$T = \underline{m_1 a + m_1 g}$ $\quad\quad$ $T = \underline{-m_2 a + m_2 g}$

$$m_1 a + m_1 g = -m_2 a + m_2 g$$

common factor.

$$m_1 a + m_2 a = m_2 g - m_1 g$$

$$a(m_1 + m_2) = g(m_2 - m_1)$$

$$\Rightarrow \quad a = \frac{g(m_2 - m_1)}{m_1 + m_2}$$

$$a = \frac{9.8(17 - 8.5)}{8.5 + 17}$$

$$a = \frac{83.3}{25.5}$$

magnitude

$$\Rightarrow \quad a = 3.3\,m/s^2$$

$\therefore$ $m_1 = 8.5\ kg$ $\quad\quad$ $a = 3.3\ m/s^2$ up

$m_2 = 17\ kg$ $\quad\quad$ $a = 3.3\ m/s^2$ down

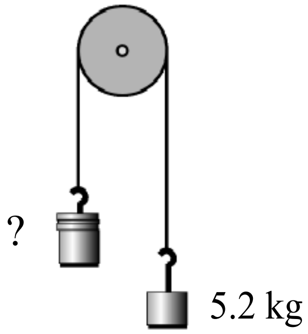
$$T = W_1 + m_1 a$$

$$T = 8.5(9.8) + 8.5(3.3)$$

$$T = 1.1 \times 10^2\ N$$

Sample Problem

The smaller mass on an Atwood machine is 5.2 kg.



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b) Find the tension in the rope.

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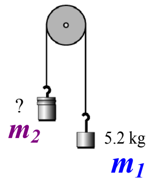

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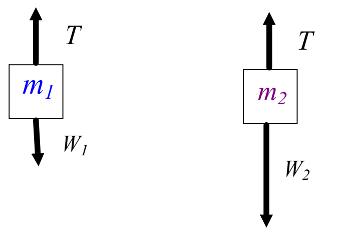
a)



$T + (-W_1) = m_1 a$ | $T + (-W_2) = -m_2 a$

$T = m_1 a + W_1$ | $T = -m_2 a + W_2$

$T = m_1 a + m_1 g$ | $T = -m_2 a + m_2 g$

$$m_1 a + m_1 g = -m_2 a + m_2 g$$

$$m_1 a + m_1 g = m_2(-a + g)$$

$$m_2 = \frac{m_1 a + m_1 g}{(-a + g)}$$

$$m_2 = \frac{5.2(4.6) + 5.2(9.8)}{-4.6 + 9.8}$$

$$m_2 = 14 \text{ kg}$$

b)

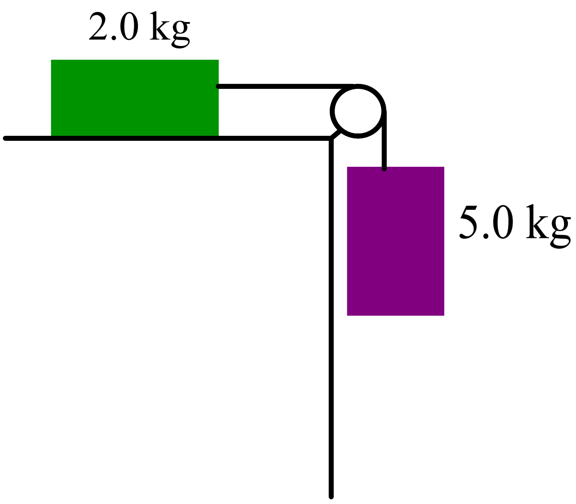
$T = m_1 a + m_1 g$

$T = 5.2(4.6) + 5.2(9.8)$

$T = 75 \text{ N}$

Sample Problem

A 2.0 kg block and a 5.0 kg block are connected as shown. Determine the magnitude of the tension in the rope connecting the two blocks. Assume the horizontal surface is frictionless.



Answer

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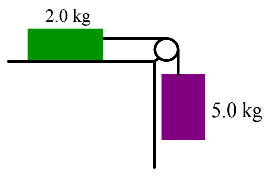
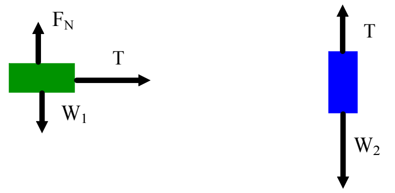




$$T = m_1 a$$

$$T + (-W_2) = -m_2 a$$
$$T = -m_2 a + W_2$$
$$T = -m_2 a + m_2 g$$

$$m_1 a = -m_2 a + m_2 g$$
$$m_1 a + m_2 a = m_2 g$$
$$a(m_1 + m_2) = m_2 g$$

$$a = \frac{m_2 g}{m_1 + m_2}$$
$$a = \frac{5.0(9.8)}{2.0 + 5.0}$$
$$a = \frac{49}{7.0}$$
$$a = 7.0\ \frac{m}{s^2}$$

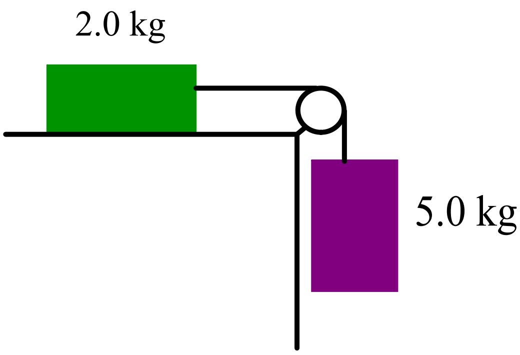
m_1 : $7.0\ \frac{m}{s^2}$, right
m_2 : $7.0\ \frac{m}{s^2}$, down

$\therefore$ $T = m_1 a$
$T = 2.0(7.0)$
$T = 14\ N$

The tension in the rope is 14 N.

Sample Problem

A 2.0 kg block and a 5.0 kg block are connected as shown. Determine the magnitude of the tension in the rope connecting the two blocks. The coefficient of kinetic friction is 0.12.



Answer

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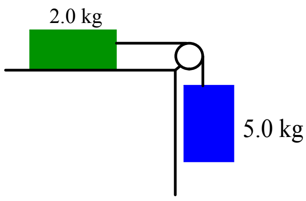
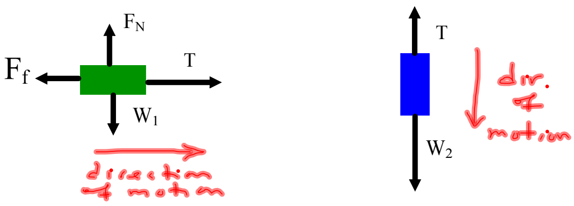




$$T + (-F_f) = m_1 a$$
$$T = m_1 a + F_f$$
$$T = m_1 a + \mu F_N$$
$$T = m_1 a + \mu m_1 g$$

$$T + (-W_2) = -m_2 a$$
$$T = -m_2 a + W_2$$
$$T = -m_2 a + m_2 g$$

$$m_1 a + \mu m_1 g = -m_2 a + m_2 g$$
$$m_1 a + m_2 a = m_2 g - \mu m_1 g$$
$$a(m_1 + m_2) = m_2 g - \mu m_1 g$$
$$a = \frac{m_2 g - \mu m_1 g}{m_1 + m_2}$$

$$a = \frac{5.0(9.8) - 0.12(2.0)(9.8)}{2.0 + 5.0}$$
$$a = \frac{49 - 2.352}{7.0}$$
$$a = 6.6\ m/s^2$$

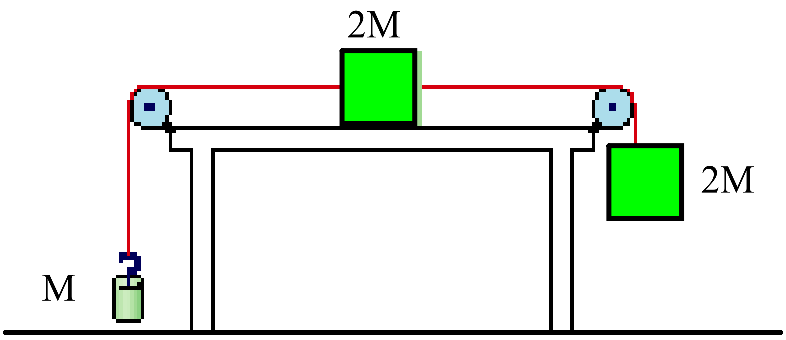
$$\boxed{\begin{array}{l} m_1 : 6.6\ m/s^2,\ \text{right} \\ m_2 : 6.6\ m/s^2,\ \text{down} \end{array}}$$

$$\therefore\ T = m_1 a + \mu m_1 g$$
$$T = 2.0(6.6) + 0.12(2.0)(9.8)$$
$$T = 16\ N$$

The tension in the rope is 16N.

High Falutin' Pulley Problem

Three blocks are released from rest and accelerate at the rate of 1.5 m/s^2. If M = 2.0 kg, what is the magnitude of the force of friction that acts on the block that slides across the table?



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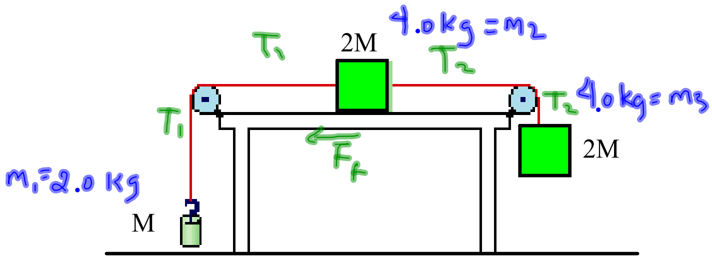


$F_{net} = m_1 a$

$T_1 + (-w_1) = m_1 a$

$T_1 = m_1 a + m_1 g$

$T_1 = 2.0(1.5) + 2.0(9.8)$

$T_1 = 22.6$ N

$F_{net} = -m_3 a$

$T_2 + (-w_3) = -m_3 a$

$T_2 = -m_3 a + m_3 g$

$T_2 = -4.0(1.5) + 4.0(9.8)$

$T_2 = 33.2$ N

$F_{net} = m_2 a$

$(-T_1) + (F_f) + T_2 = m_2 a$

$(-T_1) + T_2 - m_2 a = F_f$

$-22.6 + 33.2 - (4.0)(1.5) = F_f$

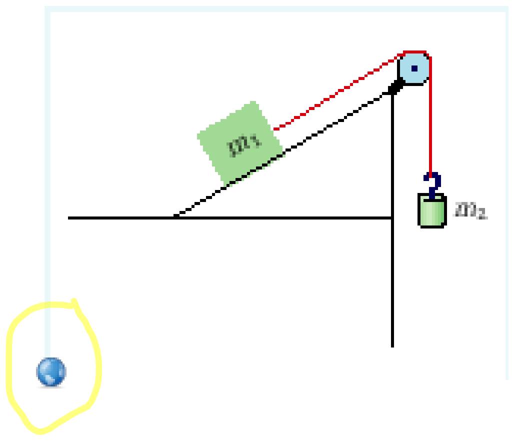
$4.6 \text{ N} = F_f$

The magnitude of the force of friction is 4.6 N.

Inclined Plane-Pulley Problem

A block of mass 3.20 kg on a frictionless inclined plane of angle 30.0° is connected by a cord over a massless, frictionless pulley to a second block of mass 2.30 kg hanging vertically. What is:

a) the magnitude of the acceleration of each block?
b) the direction of the acceleration of the hanging block?
c) the magnitude of the tension in the cord?



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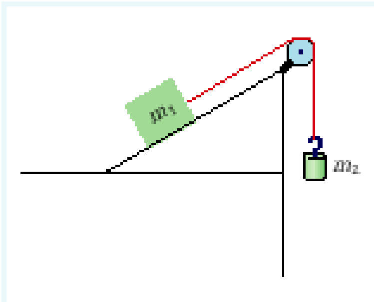


$(-F) + T = m_1 a$
$T = m_1 a + F_c$
$T = m_1 a + w_1 \sin\theta$
$T = m_1 a + m_1 g \sin\theta$

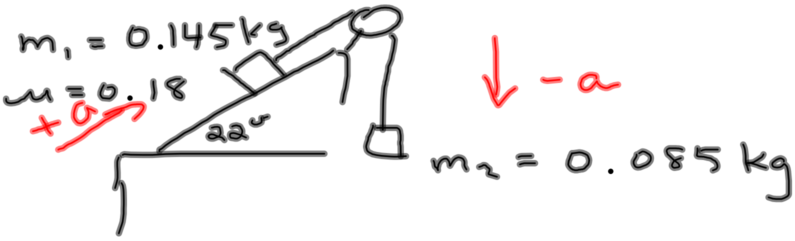
$T + (-w_2) = -m_2 a$
$T = -m_2 a + w_2$
$T = -m_2 a + m_2 g$

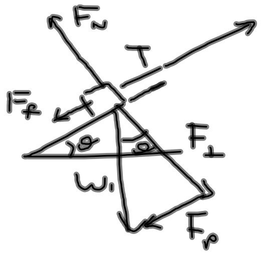
$m_1 a + m_1 g \sin\theta = -m_2 a + m_2 g$
$m_1 a + m_2 a = m_2 g - m_1 g \sin\theta$
$a(m_1 + m_2) = m_2 g - m_1 g \sin\theta$
$a = \dfrac{m_2 g - m_1 g \sin\theta}{m_1 + m_2}$

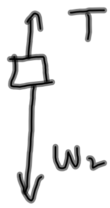
$a = \dfrac{2.3(9.8) - 3.2(9.8)\sin 30°}{3.20 + 2.30}$
$a = \dfrac{22.54 - 15.68}{5.5}$
$a = 1.25\ m/s^2$

T = 19.7 N

Question 27, pg. 489







$$T + (-F_f) + (-F_p) = m_1 a$$
$$T = m_1 a + F_f + F_p$$
$$T = m_1 a + \mu F_N + W_1 \sin\theta$$
$$T = m_1 a + \mu F_\perp + m_1 g \sin\theta$$
$$T = m_1 a + \mu W_1 \cos\theta + m_1 g \sin\theta$$
$$T = m_1 a + \mu m_1 g \cos\theta + m_1 g \sin\theta$$

$$T + (-W_2) = -m_2 a$$
$$T = -m_2 a + W_2$$
$$T = -m_2 a + m_2 g$$

$$m_1 a + \mu m_1 g \cos\theta + m_1 g \sin\theta = -m_2 a + m_2 g$$
$$\mu m_1 g \cos\theta + m_1 g \sin\theta - m_2 g = -m_2 a - m_1 a$$
$$\mu m_1 g \cos\theta + m_1 g \sin\theta - m_2 g = a(-m_2 - m_1)$$
$$\frac{\mu m_1 g \cos\theta + m_1 g \sin\theta - m_2 g}{-m_2 - m_1} = a$$

$$0.28\,\frac{m}{s^2} = a$$

$$T = -m_2 a + m_2 g$$
$$T = -(0.085)(0.2765) + 0.085(9.8)$$
$$T = 0.81\,N$$

October 6, 2009

A more complicated friction inclined plane problem

Bell Work

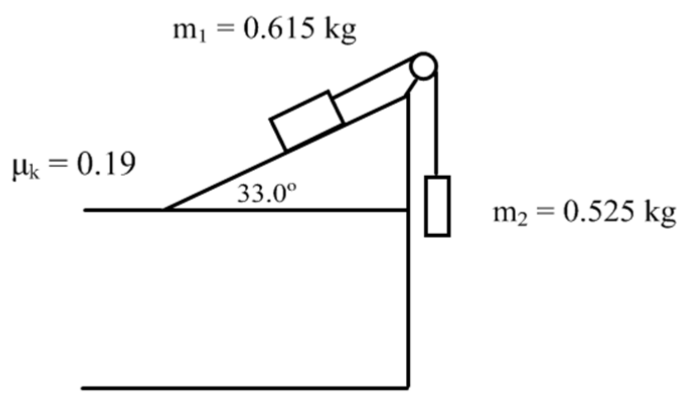
Problem to Solve

You have ~ 10 minutes to answer this question.

Pick up the handout from the green desk at the front of the room. Solve this problem in your Hilroy scribbler.

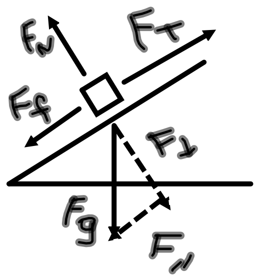
Reminder
Quiz TOMORROW!!!
Inclined Planes and Pulleys

What will be the acceleration of each object once they are set into motion?



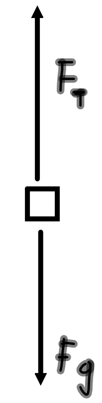
m_1

acceleration up incline



m_2

accel down



$$F_{net} = F_T - F_f - F_{\parallel}$$

$$m_1 a = F_T - \mu F_N - F_g \sin\theta$$

$$m_1 a = F_T - \mu(F_\perp) - m_1 g \sin\theta$$

$$m_1 a = F_T - \mu(F_g \cos\theta) - m_1 g \sin\theta$$

$$m_1 a = F_T - \mu(m_1 g \cos\theta) - m_1 g \sin\theta$$

$$m_1 a + \mu m_1 g \cos\theta + m_1 g \sin\theta = F_T$$

$$F_{Net} = -m_2 a$$

$$F_T - F_g = -m_2 a$$

$$F_T - m_2 g = -m_2 a$$

$$F_T = -m_2 a + m_2 g$$

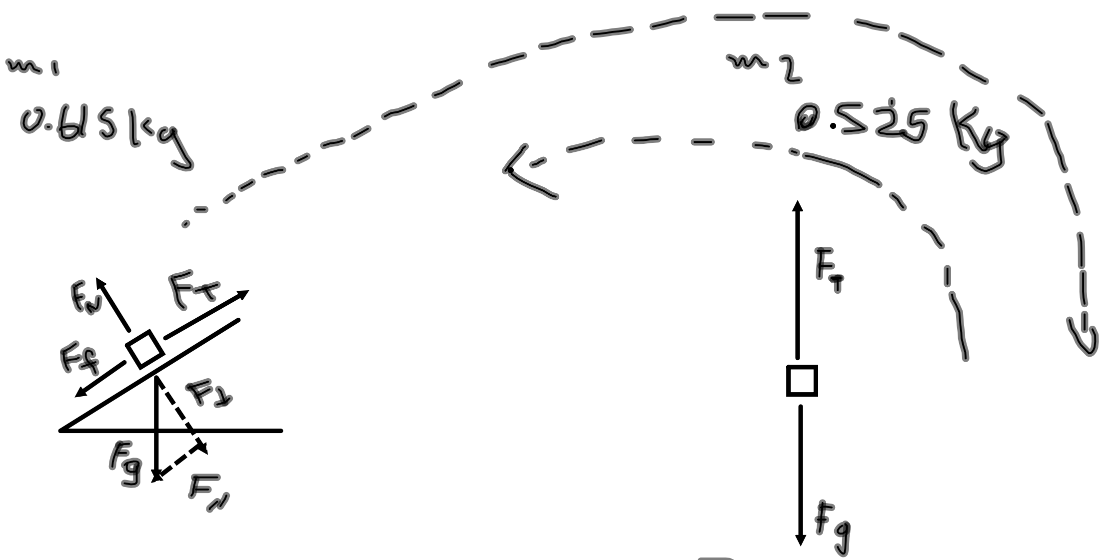
equal

$$\therefore m_1 a + \mu m_1 g \cos\theta + m_1 g \sin\theta = -m_2 a + m_2 g$$

$$0.615a + 0.19(0.615)(9.8)\cos 33.0^\circ + 0.615(9.8)\sin 33.0 = -0.525a + 0.525(9.8)$$

$$0.615a + 0.525a = 5.145 - 0.9604 - 3.2825$$

$$a = \frac{0.9021}{1.14} = 0.80 \text{ m/s}^2$$



$F_\perp = F_g \cos 33^\circ$
$= m_1 g \cos 33^\circ$
$= 5.05N$

$F_\parallel = F_g \sin 33^\circ$
$= m_1 g \sin 33^\circ$
$= 3.28N$

$F_g = m_2 g$
$= 0.525(9.8)$
$= 5.15N$

$F_N = F_\perp = 5.05N$

$F_f = \mu F_N$
$= (0.19)(5.05N)$
$= 0.96N$

$F_{Net} = F_T + F_g$

$F_T = F_g$

$F_{Net} = F_T - (F_\parallel + F_f)$
$= F_T - (3.28 + 0.96)$
$= 5.15 - (4.24)$
$= 0.91N$

All the objects will move together as they are all connected, so we must consider the total mass of all the objects to resolve the acceleration of each object.

$F_{Net} = m_T a$

$0.91N = (m_1 + m_2)a$

$a = \dfrac{0.91N}{1.14\,kg} = 0.80\,m/s^2$

Incline Plane with Friction and Tension: physics challenge problem

Bell Work

October 5, 2009

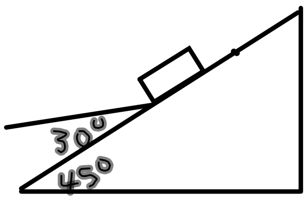
Problem to Solve

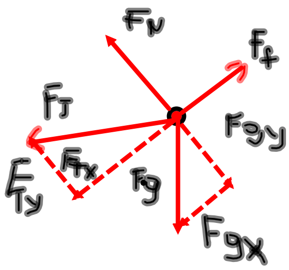
You have ~ 10 minutes to solve this problem in your Hilroy scribbler.

A box is pulled down a 45.0^{o} ramp by a rope attached to the box that makes an angle of 30.0^{o} with the ramp. If a tension of F_T causes the box to move at a constant velocity, what is the coefficient of friction for the box sliding down the ramp? *Draw a neatly labeled FBD for this scenario.*

Quiz - Wednesday, October 7:
Inclined Planes and Pulleys

A box is pulled down a 45.0° ramp by a rope attached to the box that makes an angle of 30.0° with the ramp. If a tension of F_T causes the box to move at a constant velocity, what is the coefficient of friction for the box sliding down the ramp? *Draw a neatly labeled FBD for this scenario.*





$F_{Tx} = F_T \cos 30^\circ$

$F_{Ty} = F_T \sin 30^\circ$

$F_{gx} = mg \sin 45^\circ$

$F_{gy} = mg \cos 45^\circ$

Perpendicular Forces

$F_{net} = 0$

$0 = F_{Ty} + F_N - F_{gy}$

$F_N = mg \cos 45^\circ - F_T \sin 30^\circ$

Parallel Forces(using down the ramp as positive)

$F_{Net} = 0$

$0 = F_{Tx} + F_{gx} - F_f$

$F_f =$

$$\mu = \frac{F_f}{F_N} = \frac{mg \sin 45 + F_T \cos 30}{mg \cos 45 - F_T \sin 30}$$

$=$

Attachments

- Levers and Pulleys - Build a Rube Golberg Machine
- Applet - inclined Plane
- Attwood Machine Part 2
- A more complicated friction inclined plane problem
- Incline Plane with Friction and Tension: physics challenge problem