5.4 -5.6

Optimization Problems

5.4 Optimization Problems I: Creating the Model

Warm Up!

Write an appropriate inequality equation for the following situations

1. A store sells two brands of TVs. Brand A sells at least twice as much as brand B.

2. A school is organizing a track and field meet. There will be no more than 250 events and no fewer than 100 events scheduled.

1. $A \geq 2B$

2. $250 \geq X \geq 100$

Optimization Problems I: Creating the Model

GOAL

Create models to represent optimization problems.

EXPLORE...

- A florist is ordering bracken fern and baby's breath for bouquets and centrepieces.
- No more than 100 stems of baby's breath will be ordered.
- More than 100 stems of bracken fern will be ordered.
- The florist has space to store no more than 250 stems, in total.
- Is each of the following a combination she can order? Explain.

Baby's Breath	Bracken Fern				
0	150				
25	25				
50	150				
100	100				
95.5	114.5				
100	150				
150	125				

Let x = Baby's Breath

Let y = Bracken Fern

$x \in w$ $Y \in w$

Linear Inequalities to be considered

1) $x \leq 100$

2) $y > 100$

3) $x + y \leq 250$

$\leq$

5.4 Optimization Problems I: Creating the Model

GOAL

Create models to represent optimization problems.

Pull for Lesson Notes

EXPLORE...

- A florist is ordering bracken fern and baby's breath for bouquets and centrepieces.
- No more than 100 stems of baby's breath will be ordered.
- More than 100 stems of bracken fern will be ordered.
- The florist has space to store no more than 250 stems, in total.
- Is each of the following a combination she can order? Explain.

Baby's Breath	Bracken Fern				
0	150				
25	25				
50	150				
100	100				
95.5	114.5				
100	150				
150	125				

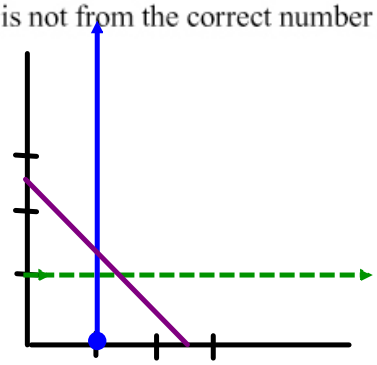
SAMPLE ANSWER

Only points (0, 150), (100, 100), (100, 150) are possible solutions because they are in the solution region and belong to the correct number set (whole numbers). Points (25, 25), (100, 100), and (150, 125) are not in the solution region, even though they have whole-number coordinates. Point (95.5, 114.5) looks like it is a possible solution. However, it is not from the correct number set, so it is not a solution.

1) $x \leq 100$

2) $y > 100$

3) $x + y \leq 250$



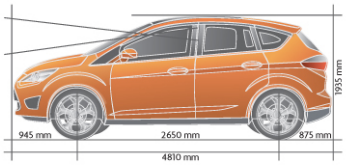
INVESTIGATE the Math

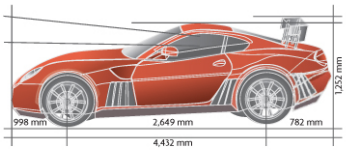
A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs $8 to make a racing car and $12 to make a sport-utility vehicle.

There are many possible combinations of racing cars and sport-utility vehicles that could be made. The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.

Pull for Lesson Notes





How can this situation be modelled?

A. What are the two variables in this situation?

B. Write a system of three linear inequalities to represent these conditions:
- the total number of racing cars that can be made
- the total number of sport-utility vehicles that can be made
- the total number of vehicles that can be made

C. What do you know about the restrictions on the domain and range of the variables? Explain.

D. Graph the system. Choose at least two points in the solution region that are possible solutions to the system.

E. What quantity in this situation needs to be minimized and maximized? Write an equation to represent how the two variables relate to this quantity.

Answers

A.

B.

C.

D.

E.

Reflecting

F. Each combination below is a possible solution to the system of linear inequalities:

i) 40 racing cars and 60 sport-utility vehicles
ii) 40 racing cars and 30 sport-utility vehicles
iii) 10 racing cars and 60 sport-utility vehicles
iv) 30 racing cars and 40 sport-utility vehicles

Use your equation from part E to calculate the manufacturing cost for each solution. What do you notice?

Answers

F. i)
ii)
iii)
iv)

Optimization Problems

Optimization problems involve finding the best possible solution in order to maximize or minimize a quantity following a set of guidelines or conditions.

Vocabulary to know: (p.244)
constraint
objective function
feasible region

constraint - a limiting condition of the optimization problem being modelled, represented by a linear inequality.

objective function - the equation that represents the relationship between the two variables in the system of linear inequalities and the quantity to be optimized.

feasible region - the solution region for a system of linear inequalities that is modelling an optimization problem

One solution in the feasible region will represent a maximum, and another a minimum.

Steps to Solving an Optimization Problem

1. Identify the quantity to be optimized (maximized or minimized). Look for key words, such as ***maximize*** or ***minimize***, ***largest*** or ***smallest***, and ***greatest*** or ***least***.

2. ***Define the variables*** that affect the quantity to be optimized. ***Identify any restrictions*** on these variables.

3. ***Write a system of linear inequalities*** to describe all constraints of the problem. ***Graph the system***.

4. ***Write an objective function*** to represent the relationship between the variables and the quantity to be optimized.

5. ***Determine the feasible region*** (possible solutions region).

6. The optimal solutions will be represented by the points of intersection of the boundaries of the feasible region, known as the vertices. ***Evaluate the objective function by substituting the values of the coordinates of each vertex.***

7. ***Verify*** that ***the solution(s)*** satisfies the constraints of the problem situation.

EXAMPLE 1 Creating a model for an optimization problem with whole-number variables

Three teams are travelling to a basketball tournament in cars and minivans.

- Each team has no more than 2 coaches and 14 athletes.
- Each car can take 4 team members, and each minivan can take 6 team members.
- No more than 4 minivans and 12 cars are available.

The school wants to know the combination of cars and minivans that will require the minimum and maximum number of vehicles.

Steps to Solving an Optimization Problem

1. Identify the quantity to be optimized (maximized or minimized). Look for key words, such as ***maximize*** or ***minimize***, ***largest*** or ***smallest***, and ***greatest*** or ***least***.

   number of vehicles

2. ***Define the variables*** that affect the quantity to be optimized. ***Identify any restrictions*** on these variables.

   let x = number of cars; must be whole numbers
   let y = number of minivans; must be whole numbers

3. ***Write a system of linear inequalities*** to describe all constraints of the problem. ***Graph the system***. (on next page)

   number of cars available: $x \leq 12$
   number of minivans available: $y \leq 4$
   maximum number of team members = 16 x 3 teams = 48
   so travel arrangements for team members: $4x + 6y \leq 48$

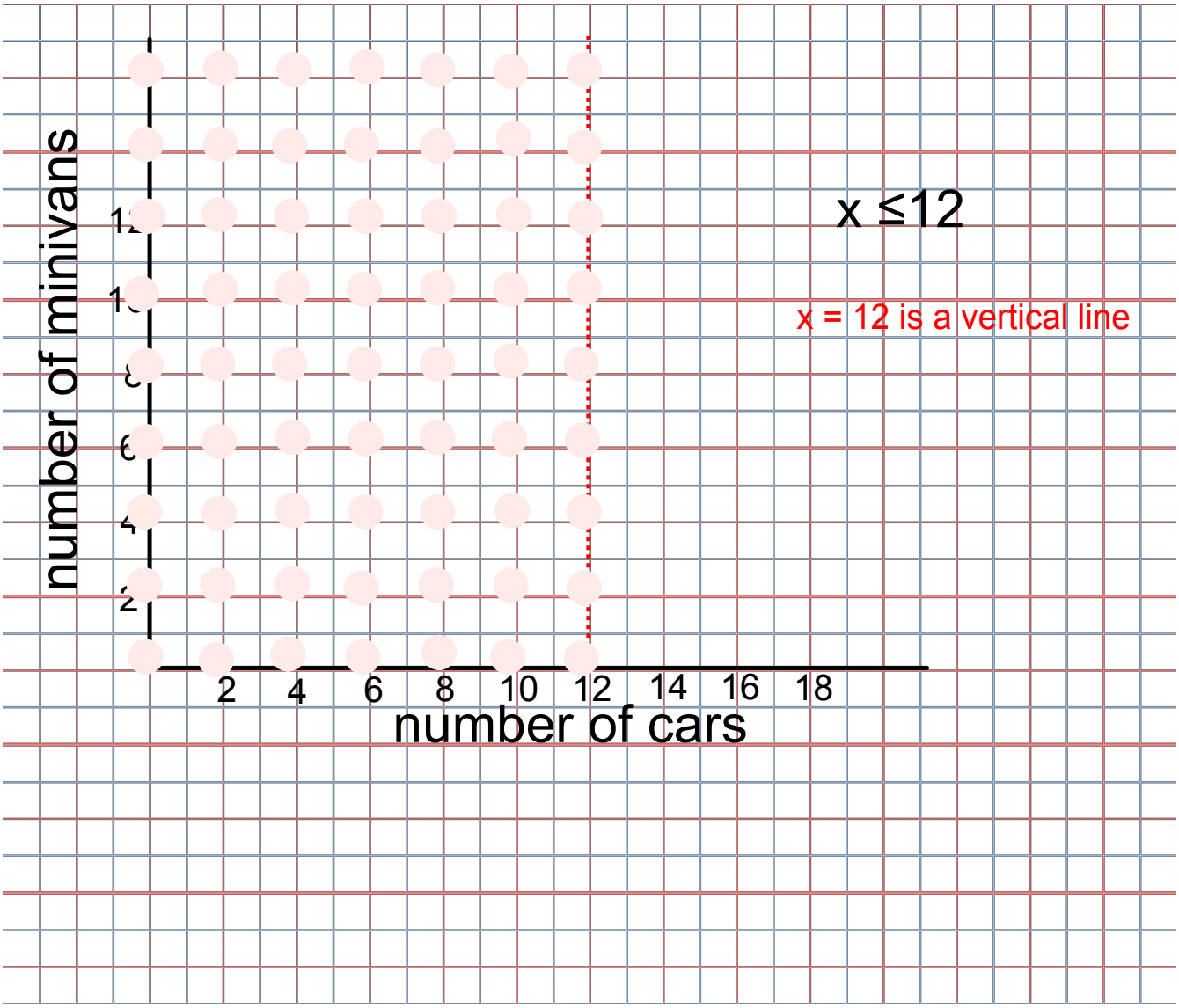
4. ***Write an objective function*** to represent the relationship between the variables and the quantity to be optimized.

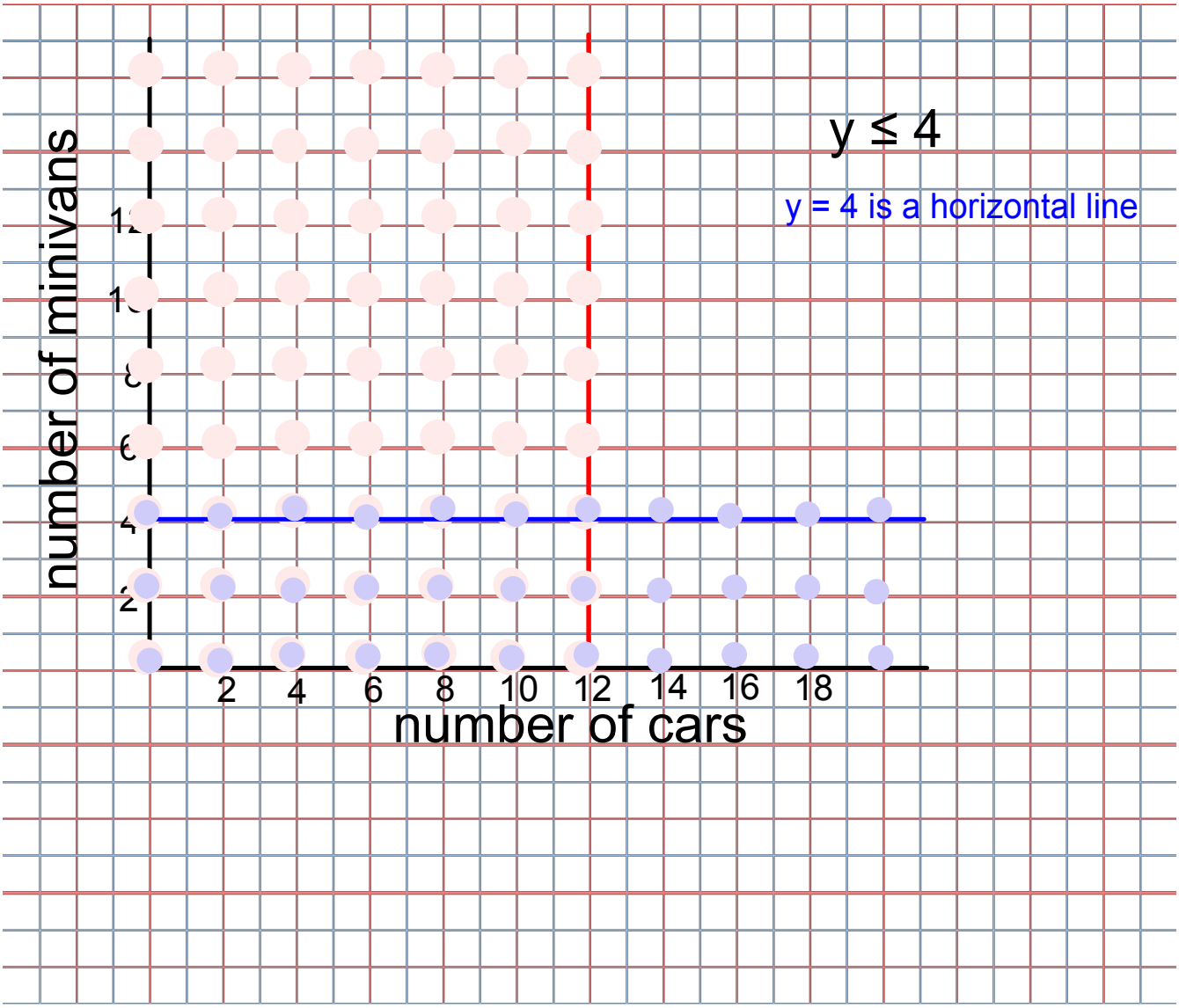
   V = total number of vehicles, so
   $V = x + y$

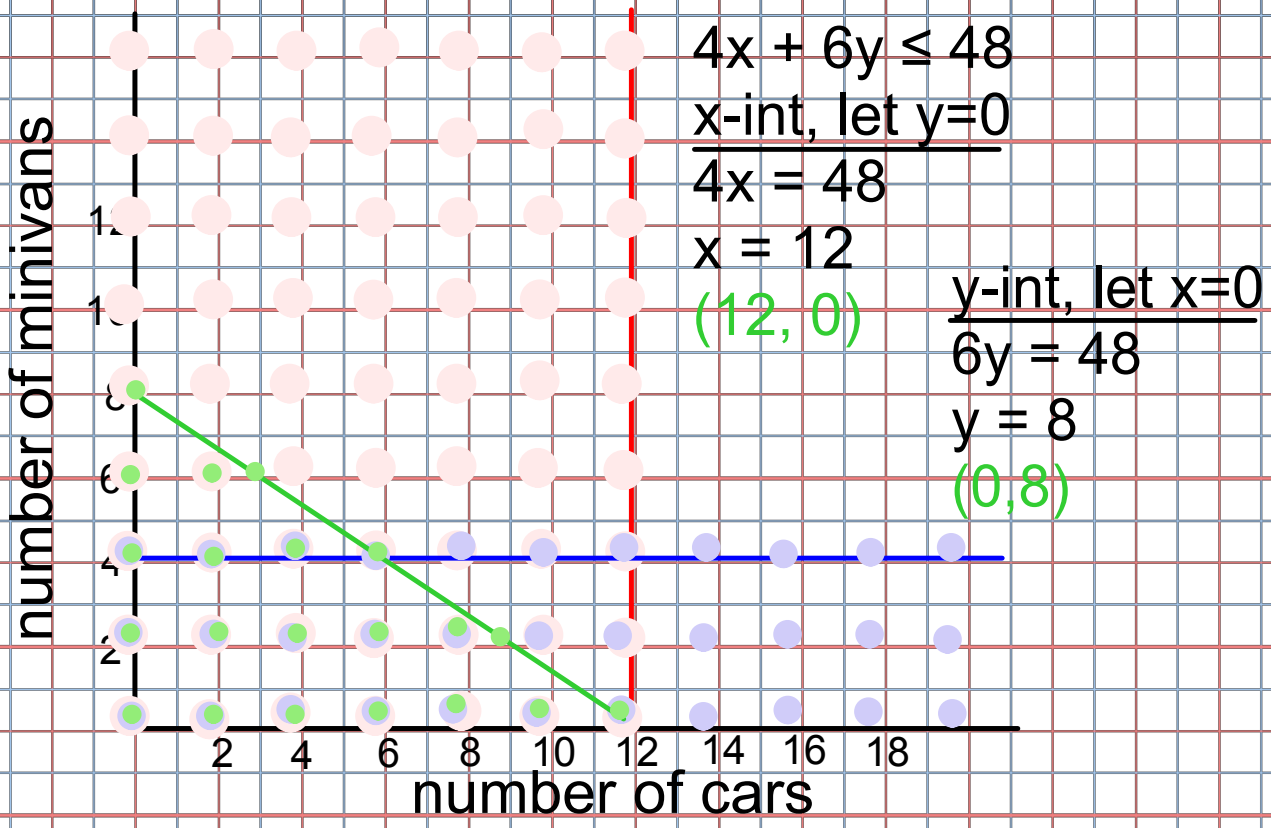
5. ***Determine the feasible region*** (possible solutions region).

6. The optimal solutions will be represented by the points of intersection of the boundaries of the feasible region, known as the vertices. ***Evaluate the objective function by substituting the values of the coordinates of each vertex.***

7. ***Verify*** that ***the solution(s)*** satisfies the constraints of the problem situation.


number of minivans
number of cars
2
4
6
8
10
12
14
16
18
x ≤12
x = 12 is a vertical line


y ≤ 4
y = 4 is a horizontal line
number of minivans
number of cars
2
4
6
8
10
12
14
16
18



$4x + 6y \leq 48$

x-int, let y=0

$4x = 48$

$x = 12$

(12, 0)

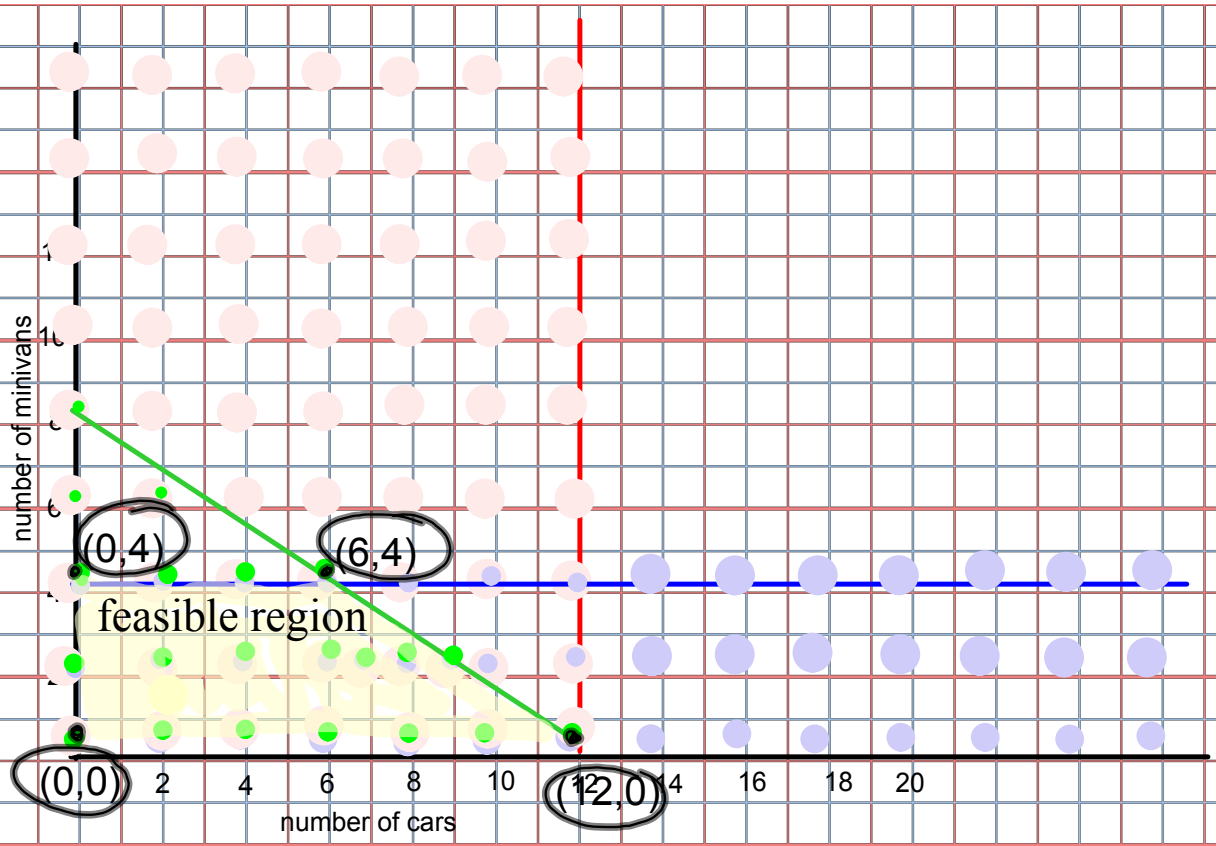
y-int, let x=0

$6y = 48$

$y = 8$

(0,8)

Where is the region of overlap? This becomes the feasible region.



Substitute coordinates of each vertex into the objective function:

V = x + y	V = x + y	V = x + y	V = x + y
(0,0) 0 + 0	(0,4) 0 + 4	(6,4) 6 + 4	(12,0) 12+0
V = 0	V = 4	V = 10	V = 12

# team members: 4x+6y			
4(0)+6(0)	4(0)+6(4)	4(6)+6(4)	4(12)+6(0)
0 + 0	0 + 24	24 + 24	48 + 0
0	24	48	48

So,

- 0 vehicles could transport 0 team members.
- 4 vehicles can transport 24 team members.
- 10 vehicles can transport 48 team members.
- 12 vehicles can transport 48 team members.

Which solution is the best one? Why?

Your Turn

Suppose that the greatest number of athletes changed from 14 to 12 per team. How would Juanita's model change?

Answer

EXAMPLE 2 Creating a model for a maximization problem with positive real-number variables

A refinery produces oil and gas.

- At least 2 L of gasoline is produced for each litre of heating oil.
- The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
- Gasoline is projected to sell for $1.10 per litre. Heating oil is projected to sell for $1.75 per litre.

The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.

Umberto's Solution

Let h represent the number of litres of heating oil.
Let g represent the number of litres of gasoline.

I knew that this is an optimization problem because the total revenue has to be maximized.

The two variables in the problem are the volume of heating oil and the volume of gasoline, both in litres. Litres are measured using positive real numbers.

Restrictions:
$h \geq 0$ and $g \geq 0$, where $h \in \mathrm{R}$ and $g \in \mathrm{R}$

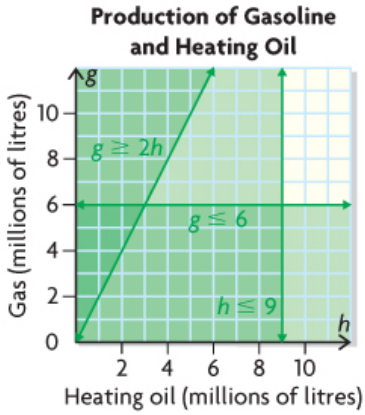
Constraints:
Ratio of gasoline produced to oil produced:
$g \geq 2h$
Amount of gasoline that can be produced:
$g \leq 6\,000\,000$
Amount of oil that can be produced:
$h \leq 9\,000\,000$

I created inequalities to represent the five constraints of the problem.

I treated the restrictions on each variable as a constraint.

Let R represent total revenue from sales of gasoline and heating oil.
Objective function to maximize:
$R = 1.10g + 1.75h$

I wrote an objective function to represent the relationship between the two variables (volume of heating oil and volume of gasoline) and the quantity to be maximized (total revenue).



I graphed the system of inequalities in the 1st quadrant because of the restrictions on the variables. The feasible region is a right triangle and includes all points on its boundaries.

I think I can use the objective function to determine which point in the feasible region represents the combination of oil and gas that will result in the maximum revenue, but I am not sure how yet.

EXAMPLE 2 Creating a model for a maximization problem with positive real-number variables

A refinery produces oil and gas.

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- The refinery can produce up to 9 million litres of heating oil and 6 million litres of gasoline each day.
- Gasoline is projected to sell for $1.10 per litre. Heating oil is projected to sell for $1.75 per litre.

The company needs to determine the daily combination of gas and heating oil that must be produced to maximize revenue. Create a model to represent this situation.

Your Turn

Suppose that the refinery produced at least 2 L of heating oil for each litre of gasoline. Discuss with a partner how Umberto's model would change.

Answer

In Summary

Key Ideas

- To solve an optimization problem, you need to determine which combination of values of two variables results in a maximum or minimum value of a related quantity.
- When creating a model, the first step is to represent the situation algebraically. An algebraic model includes these parts:
  - a defining statement of the variables used in your model
  - a statement describing the restrictions on the variables
  - a system of linear inequalities that describes the constraints
  - an objective function that shows how the variables are related to the quantity to be optimized
- The second step is to represent the system of linear inequalities graphically.
- In optimization problems, any restrictions on the variables are considered constraints. For example, if you are working with positive real numbers, $x \geq 0$ and $y \geq 0$ are constraints and should be included in the system of linear inequalities.

Need to Know

- You can create a model for an optimization problem by following these steps:

  Step 1. Identify the quantity that must be optimized. Look for key words, such as *maximize* or *minimize*, *largest* or *smallest*, and *greatest* or *least*.

  Step 2. Define the variables that affect the quantity to be optimized. Identify any restrictions on these variables.

  Step 3. Write a system of linear inequalities to describe all the constraints of the problem. Graph the system.

  Step 4. Write an objective function to represent the relationship between the variables and the quantity to be optimized.

Warm Up!

Sophie has two summer jobs.

- **She works no more than a total of 32 hours a week. Both jobs allow her to have flexible hours, but in whole hours only.**
- **At one job, Sophie works no less than 12 hours and earns $8.75/h.**
- **At the other job, Sophie works no more than 24 hours and earns $9.00/h.**

What combination of numbers of hours will allow her to maximize her earnings? What can she expect to earn?

Warm Up!

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What combination of numbers of hours will allow her to maximize her earnings? What can she expect to earn?

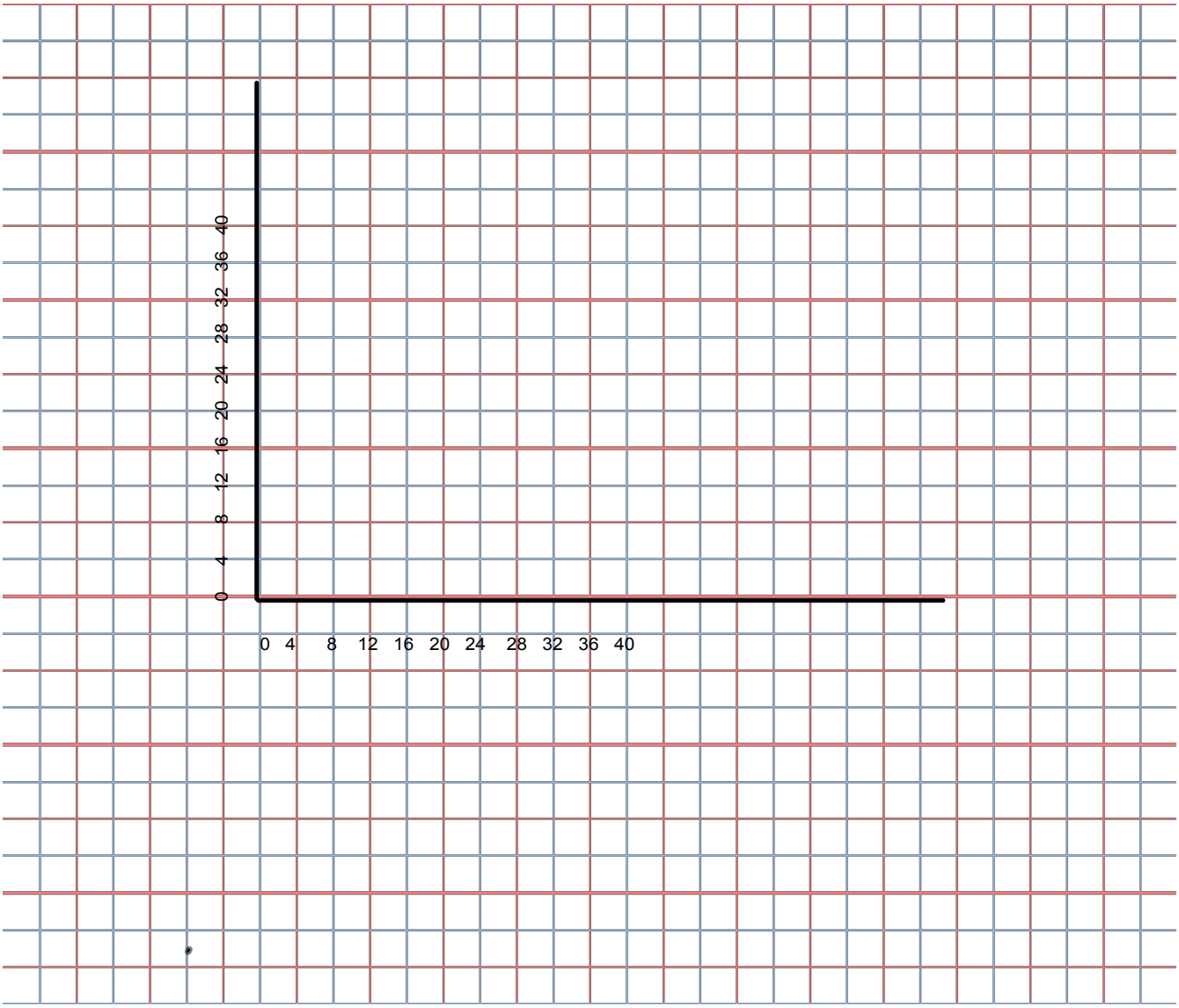
x = # Job 1
y = Job 2

$x + y \leq 32$

$x \geq 12$

$y \leq 24$

$8.75x + 9.00y = E$


0 4 8 12 16 20 24 28 32 36 40
0 4 8 12 16 20 24 28 32 36 40

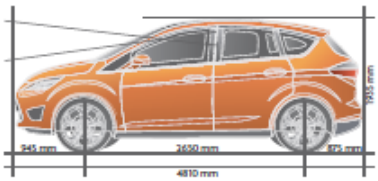
5.5 Optimization Problems II: Exploring Solutions

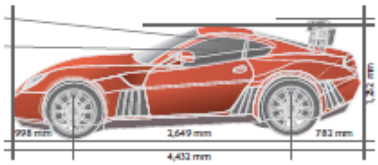
Remember the Vehicle Problem

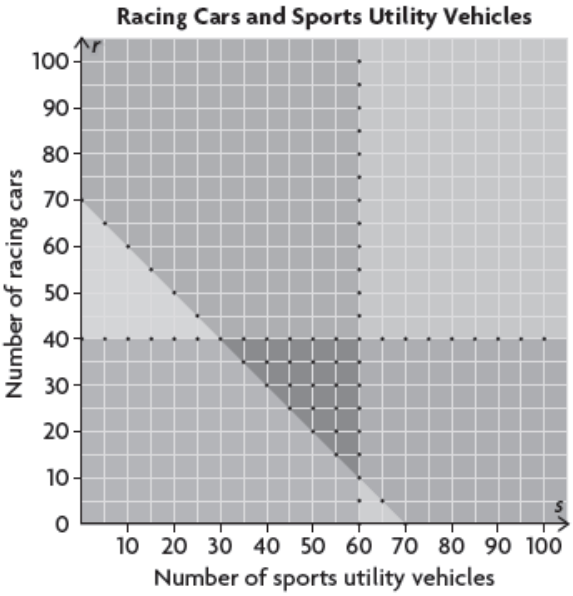
A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs $8 to make a racing car and $12 to make a sport-utility vehicle.

There are many possible combinations of racing cars and sport-utility vehicles that could be made. The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.







How was each linear inequality derived ?

How did you know that the domain and range of the variables is a set of whole numbers?

How did the graph help solve the problem?

5.5 Optimization Problems II: Exploring Solutions

Pull for Lesson Notes

GOAL

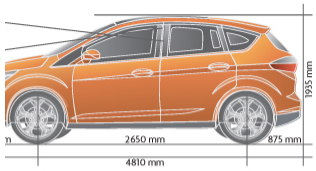
Explore the feasible region of a system of linear inequalities.

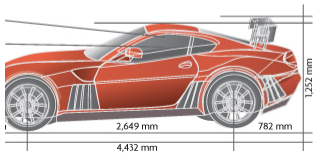
EXPLORE the Math

A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs \$8 to make a racing car and \$12 to make a sport-utility vehicle.

There are many possible combinations of racing cars and sport-utility vehicles that could be made. The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.



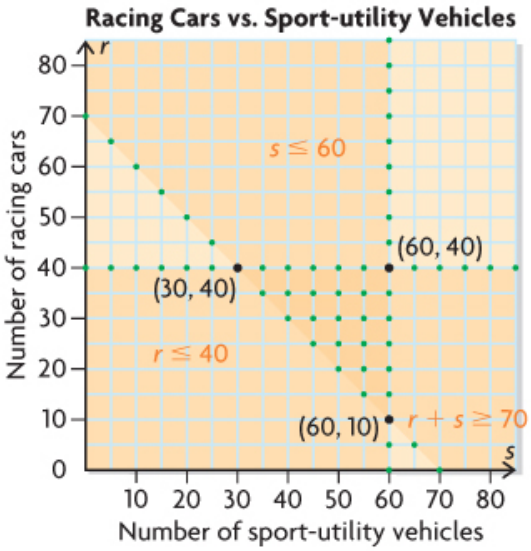


The following model represents this situation. The feasible region of the graph represents all the possible combinations of racing cars (r) and sport-utility vehicles (s).

Variables:
Let s represent the number of sport-utility vehicles.
Let r represent the number of racing cars.
Let C represent the cost of production.

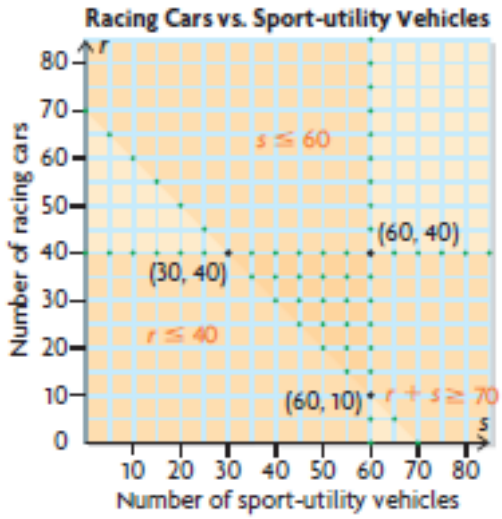
Restrictions:
$s \in W, r \in W$

Constraints:
$s \geq 0$
$r \geq 0$
$r \leq 40$
$s \leq 60$
$r + s \geq 70$



Objective function to optimize:
$C = 12s + 8r$

? How can you use patterns in the feasible region to predict the combinations of sport-utility vehicles and racing cars that will result in the minimum and maximum values of the objective function?



? How can you use patterns in the feasible region to predict the combinations of sport-utility vehicles and racing cars that will result in the minimum and maximum values of the objective function?

Sample Solution

Value of $C = 8r + 12s$

- as s increases: 720 at (40, 30), 840 at (50, 30), 960 at (60, 30)
- as r increases: 860 at (45,40), 780 at (45, 30) 740 at (45, 25)
- in the middle of the solution region: 820 at (45, 35), 880 at (50, 35), 840 at (50, 30)
- at the corners of the solution region: 800 at (60, 10), 1040 at (60, 40), 680 at (30, 40)

Reflecting

A. With a partner, discuss the pattern in the value of C throughout the feasible region. Is the pattern what you expected? Explain.

B. As you move from left to right across the feasible region, what happens to the value of C?

C. As you move from the bottom to the top of the feasible region, what happens to the value of C?

D. What points in the feasible region result in each **optimal solution**?
   i) the maximum possible value of C
   ii) the minimum possible value of C

E. Explain how you could verify that your solutions from part D satisfy each constraint in the model.

optimal solution

A point in the solution set that represents the maximum or minimum value of the objective function.

Answers

A.

B.

C.

D. i)

ii)

E. i)

ii)

In Summary

Key Ideas

- The value of the objective function for a system of linear inequalities varies throughout the feasible region, but in a predictable way.
- The optimal solutions to the objective function are represented by points at the intersections of the boundaries of the feasible region. If one or more of the intersecting boundaries is not part of the solution set, the optimal solution will be nearby.

Need to Know

- You can verify each optimal solution to make sure it satisfies each constraint by substituting the values of its coordinates into each linear inequality in the system.
- The intersection points of the boundaries are called the vertices, or corners, of the feasible region.

Warm up

Ed found spiders and crickets in his storage room.

- There were 20 or fewer spiders 20 or more crickets.
- There were 45 or fewer crickets and spiders, in total. Spiders hav

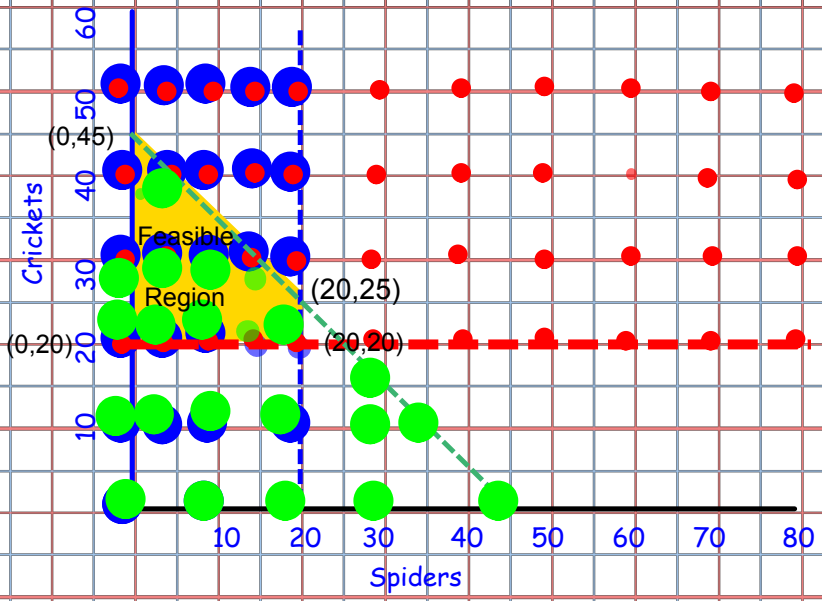
8 legs, and crickets have 6 legs.

a) What combination of spiders and crickets would have the greates number of legs?

b) What combination would have the least number of legs?

x = spiders
y= crickets

$x \leq 20$
$y \geq 20$
$x+y \leq 45$
8x+6y = # Legs



8X+6Y = L
8(0) + 6(20) = 120
8(0) + 6(45) = 270
8(20) + 6(20) = 280
8(20) + 6(25) = 310

5.6 Optimization Problems III: Linear Programming

GOAL

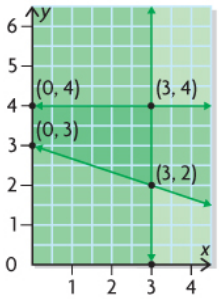
Solve optimization problems.

EXPLORE...

Pull for Lesson Notes

- The following system of linear inequalities has been graphed below:

System of linear inequalities:
$y \geq 0$
$x \geq 0$
$y \leq 4$
$x \leq 3$
$3y \geq -x + 9$



a) For each objective function, what points in the feasible region represent the minimum and maximum values?
i) $T = 5x + y$
ii) $T = x + 5y$
b) What do you notice about the optimal points for the two objective functions? Why do you think this happened?

SAMPLE ANSWER

a) i)

ii)

b)

LEARN ABOUT the Math

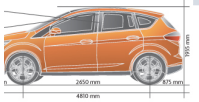
In Lesson 6.4, you created an optimization model for the problem below. In Lesson 6.5, you explored the model for optimal solutions.

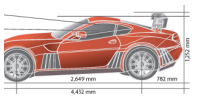
Pull for Lesson Notes

A toy company manufactures two types of toy vehicles: racing cars and sport-utility vehicles.

- Because the supply of materials is limited, no more than 40 racing cars and 60 sport-utility vehicles can be made each day.
- However, the company can make 70 or more vehicles, in total, each day.
- It costs $8 to make a racing car and $12 to make a sport-utility vehicle.

There are many possible combinations of racing cars and sport-utility vehicles that could be made. The company wants to know what combinations will result in the minimum and maximum costs, and what those costs will be.





? Which combinations of the two types of vehicles will result in a minimum and a maximum production cost, and what will these costs be?

EXAMPLE 1 Solving an optimization problem by graphing an algebraic model

Ramona's Solution

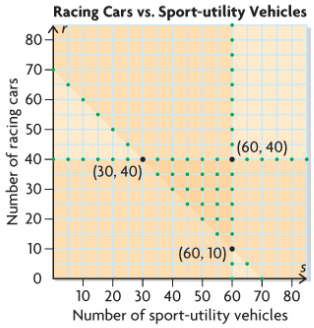
I started by creating an algebraic model to represent the situation.

Let s represent sport-utility vehicles.
Let r represent racing cars.

Restrictions on the variables:
$r \in W, s \in W$

Constraints:
$r \geq 0$
$s \geq 0$
$r \leq 40$
$s \leq 60$
$r + s \geq 70$

Let C represent the total production cost.
Objective function to optimize, at $12 per sport-utility vehicle and $8 per car:
$C = 12s + 8r$



I graphed the system of inequalities to determine the points at the vertices of the feasible region.

I knew that two of these points represent the solutions that will optimize the objective function.

I conjectured that

- the minimum solution is (30, 40) because this represents the lowest total number of vehicles (70) and there are fewer of the more expensive sport-utility vehicles than racing cars (30 versus 40).
- the maximum solution is (60, 40) because this represents the highest total number of vehicles (100) and there is a greater number of the more expensive sport-utility vehicles than racing cars (60 vs. 40).

Optimize:
$C = 12s + 8r$

If (s, r) is (60, 10),	If (s, r) is (60, 40),	If (s, r) is (30, 40),
$C = 12(60) + 8(10)$	$C = 12(60) + 8(40)$	$C = 12(30) + 8(40)$
$C = 720 + 80$	$C = 720 + 320$	$C = 360 + 320$
$C = \$800$	$C = \$1040$	$C = \$680$

To check my conjecture, I evaluated the objective function using the coordinates of each vertex of the feasible region. This enabled me to compare the production cost for each solution.

The company can minimize the production cost to $680 by making 30 sport-utility vehicles and 40 racing cars and maximize costs to $1040 by making 60 sport-utility vehicles and 40 racing cars.

My conjecture was correct.

Verify minimum (30, 40):

(s, r) is (30, 40): $r \leq 40$		(s, r) is (30, 40): $s \leq 60$		(s, r) is (30, 40): $r + s \geq 70$	
LS	RS	LS	RS	LS	RS
40	40	30	60	30 + 40	70
$40 \leq 40$		$30 \leq 60$		$70 \geq 70$	

I verified each optimal solution to make sure it satisfied every constraint in the system. Each inequality statement was valid for both optimal solutions.

Verify minimum (60, 40):

(s, r) is (60, 40): $r \leq 40$		(s, r) is (60, 40): $s \leq 60$		(s, r) is (60, 40): $r + s \geq 70$	
LS	RS	LS	RS	LS	RS
40	40	60	60	60 + 40	70
$40 \leq 40$		$60 \leq 60$		$100 \geq 70$	

Reflecting

A. How would the solution change if the cost to make a racing car was $12 and the cost to make a sport-utility vehicle was $8? How could you have predicted this?

B. How would the solution change if the cost was $10 for each vehicle? How could you have predicted that?

C. Summarize the steps you can follow when using **linear programming** to solve an optimization problem.

linear programming

A mathematical technique used to determine which solutions in the feasible region result in the optimal solutions of the objective function.

Answers

A.

B.

C.

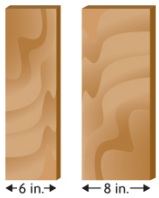
APPLY the Math

EXAMPLE 2 Creating a model for an optimization problem, and solving the problem

L&G Construction is competing for a contract to build a fence.

- The fence will be no longer than 50 yd and will consist of narrow boards that are 6 in. wide and wide boards that are 8 in. wide.
- There must be no fewer than 100 wide boards and no more than 80 narrow boards.
- The narrow boards cost $3.56 each, and the wide boards cost $4.36 each.

Determine the maximum and minimum costs for the lumber to build the fence.



Aisla's Solution

TI-*n*spire

Let n represent the number of narrow boards.
Let w represent the number of wide boards.
Let C represent the total cost of the lumber.

I began by defining the variables in this situation.

Restrictions:
$n \in W$ and $w \in W$

Since both n and w represent the number of boards needed, they must be whole numbers.

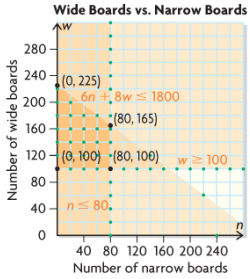
No fewer than 100 wide boards:
$w \geq 100$
No more than 80 narrow boards:
$n \leq 80$
The fence is no longer than 50 yd and is made of 6 in. narrow and 8 in. wide boards:
$6n + 8w \leq 50(36)$

Next, I created the constraints in the problem. I represented these constraints with linear inequalities.

I converted yards to inches using the rate 36 in./1 yd, since the coefficients of the other terms are in inches.

Optimization Model
Constraints:
$n \geq 0$
$w \geq 0$
$w \geq 100$
$n \leq 80$
$6n + 8w \leq 1800$
Objective function to optimize at $3.56 per narrow board and $4.36 per wide board:
$C = 3.56n + 4.36w$

I created the objective function that represents the relationship between the cost of the two widths of boards and the total cost of the lumber.



I graphed in the first quadrant because the domain and range are restricted to whole numbers. I decided to make the the number of narrow boards (n) the independent variable.

I identified the coordinates of the vertices of the feasible region.

I made these conjectures:

- Point (80, 165) will result in the maximum cost since it is farthest from both axes, which means it has large coordinate values.
- Point (0, 100) will result in the minimum cost, since one coordinate is 0 and the other coordinate is only 100.

$C = 3.56n + 4.36w$

If (n, w) is (0, 100), $C = 3.56(0) + 4.36(100)$ $C = \$436.00$	If (n, w) is (0, 225), $C = 3.56(0) + 4.36(225)$ $C = \$981.00$
If (n, w) is (80, 100), $C = 3.56(80) + 4.36(100)$ $C = \$720.80$	If (n, w) is (80, 165), $C = 3.56(80) + 4.36(165)$ $C = \$1004.20$

I substituted the values of n and w for all four vertices into the objective function to compare the cost for these solutions.

(0, 100), or no narrow boards and 100 wide boards, cost the minimum amount: $436.

(80, 165), or 80 narrow boards and 165 wide boards, cost the maximum amount: $1004.20.

My conjectures were correct.

Verify minimum (0, 100):

(n, w) is (0, 100): $n \leq 80$		(n, w) is (0, 100): $w \geq 100$		(n, w) is (0, 100): $6n + 8w \leq 1800$	
LS	RS	LS	RS	LS	RS
0	80	100	100	$6(0) + 8(100)$ 800	1800
$0 \leq 80$		$100 \geq 100$		$800 \leq 1800$	

Verify maximum (80, 165):

(n, w) is (80, 165): $n \leq 80$		(n, w) is (80, 165): $w \geq 100$		(n, w) is (80, 165): $6n + 8w \leq 1800$	
LS	RS	LS	RS	LS	RS
80	80	165	100	$6(80) + 8(165)$ 1800	1800
$80 \leq 80$		$165 \geq 100$		$1800 \leq 1800$	

I verified both solutions to make sure they satisfied every constraint in the system. Each inequality statement was valid for both optimal solutions.

APPLY the Math

EXAMPLE 2 Creating a model for an optimization problem, and solving the problem

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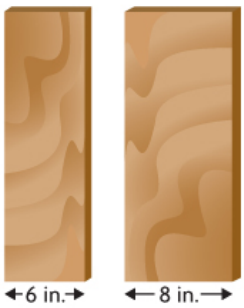
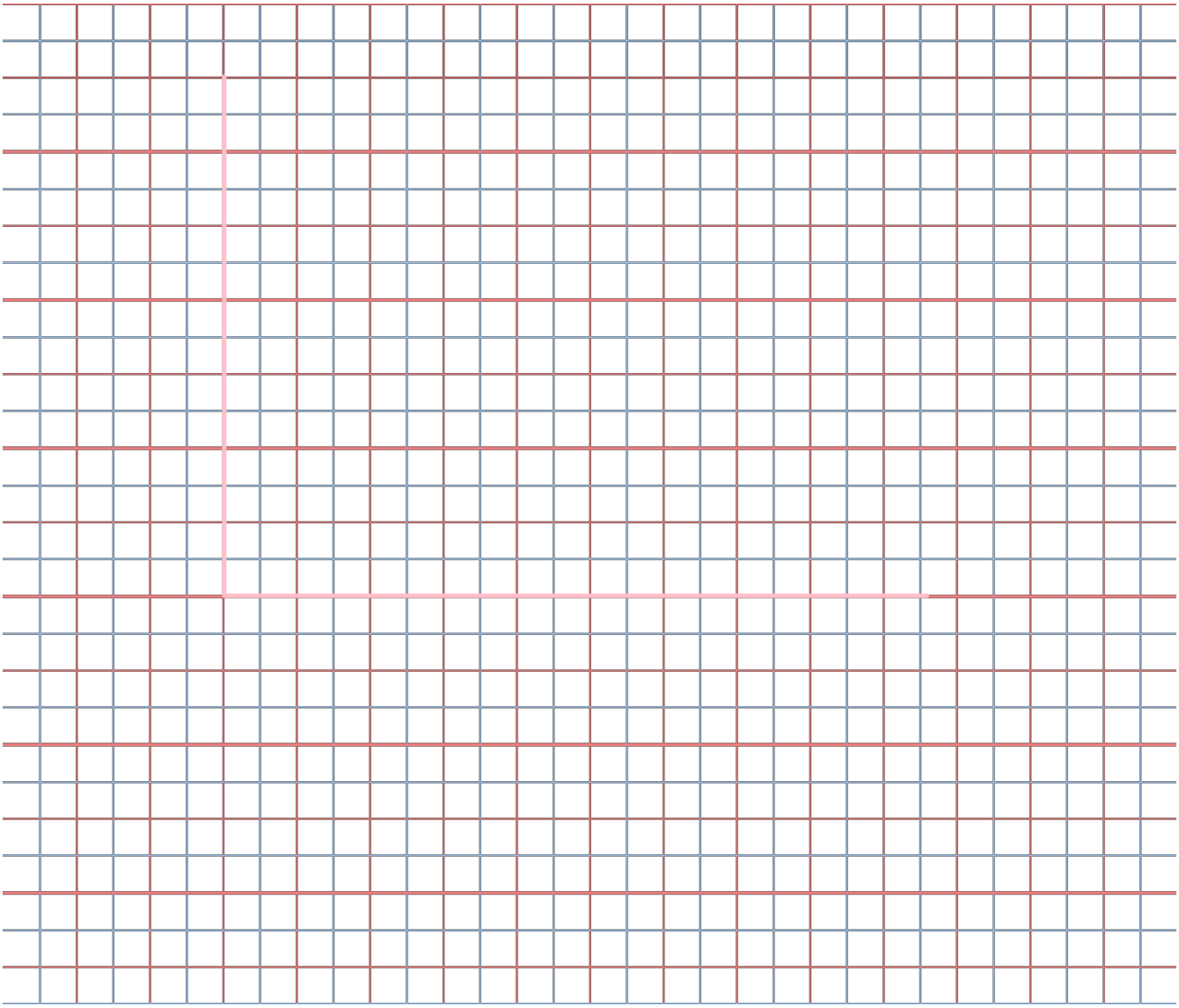


Your Turn

How would the feasible region change if 80 or more narrow boards had to be used?

Answer



In Summary

Key Idea

- To solve an optimization problem using linear programming, begin by creating algebraic and graphical models of the problem (as shown in Lesson 6.4). Then use the objective function to determine which vertex of the feasible region results in the optimal solution.

Need to Know

- The solution to an optimization problem is usually found at one of the vertices of the feasible region.
- To determine the optimal solution to an optimization problem using linear programming, follow these steps:

Step 1. Create an algebraic model that includes:
  - a defining statement of the variables used in your model
  - the restrictions on the variables
  - a system of linear inequalities that describes the constraints
  - an objective function that shows how the variables are related to the quantity to be optimized

Step 2. Graph the system of inequalities to determine the coordinates of the vertices of its feasible region.

Step 3. Evaluate the objective function by substituting the values of the coordinates of each vertex.

Step 4. Compare the results and choose the desired solution.

Step 5. Verify that the solution(s) satisfies the constraints of the problem situation.

A Saskatchewan farmer is planting wheat and barley.

- He wants to plant no more than 1000 ha altogether.
- The farmer wants at least three times as many hectares of wheat as barley.
- The yield per hectare of wheat averages 50 bushels, and the yield per hectare of barley averages 38 bushels.
- Wheat pays the farmer $5.25 per bushel, and barley pays $3.61 per bushel.

Help the farmer determine the combination of wheat and barley he should plant that will maximize revenue.

Your Turn!

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Attachments

6Ws4e1.mp4

6Ws4e2.mp4

fm6s6-p4.tns

6Ws6e2.mp4